Mathematics Class X
Chapter -4
Quadratic Equations
Module 3/3

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Learning outcomes in module 3/3 are:

- Deriving Quadratic Formula.
- Solving quadratic equation by using Quadratic Formula.
- Understanding the Nature of the Roots.

Deriving Quadratic Formula.

Consider the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$.

Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

This is same as $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

i.e, $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$

So,the roots of the given equation are the same as those of

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$,i.e, $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ --------------(i)

If $b^2 - 4ac \geq 0$,then by taking the square roots in (i) ,we get

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Therefore $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So, the roots of $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ & $\frac{-b-\sqrt{b^2-4ac}}{2a}$, if $b^2 - 4ac \geq 0$.

If $b^2 - 4ac \leq 0$, the equation will have no real roots.

Thus, if $b^2 - 4ac \geq 0$,then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

This formula for finding the roots of a quadratic equation is known as the **Quadratic Formula.**

Note: This formula was first given by an ancient Indian mathematician Sridharacharya around 1025 A.D. Therefore ,it is called as **Sridharcharya's formula** for finding roots of the quadratic equation $ax^2 + bx + c = 0$.

Example for illustrating the use of the quadratic formula.

Solve: $16x^2-24x-1=0$

Solution: Compare the given equation with $ax^2 + bx + c = 0$.

$\therefore$ a=16, b=24 & c= -1

Since $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-24) \pm \sqrt{(-24)^2-4(16)(-1)}}{2(16)} = \frac{24 \pm \sqrt{576+64}}{32} = \frac{24 \pm \sqrt{640}}{32}$

$= \frac{24 \pm 8\sqrt{10}}{32} = \frac{3 \pm \sqrt{10}}{4}$

Thus roots are $\frac{3 \pm \sqrt{10}}{4}$ i.e. $\frac{3+\sqrt{10}}{4}$, $\frac{3-\sqrt{10}}{4}$.

Nature of Roots

The roots of the quadratic equation $ax^2 + bx + c = 0 = \frac{-b \pm \sqrt{D}}{2a}$

Where $D = b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D.There are three cases-

Case-I

When D>0 i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots. i.e, $x = \frac{-b+\sqrt{D}}{2a}$ & $\frac{-b-\sqrt{D}}{2a}$

Case-II

When D = 0, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ & $\frac{-b}{2a}$

Case-III

When D<0 then there is no real roots exist.

What is the nature of quadratic equation $4x^2-12x-9 = 0$.

Solution: Here, $D = b^2 - 4ac = (-12)^2 - (-4)(9) = 144+144 = 288>0$

$\Rightarrow$Roots are real & different.

Find the value of m so that the quadratic equation $mx(x-7)+49=0$ has two equal roots.

Solution: The given equation is: $mx^2 - 7mx + 49 = 0$

For equal roots $(-7m)^2 - 4m(49) = 0$

$49m^2 - 4(49)m = 0$

$49m(m-4) = 0$

$m(m-4) = 0$

$m = 0, m = 4$

But $m = 0$ does not satisfy the given equation

Therefore $m = 4$.

Is the following situation possible?

The sum of the ages of a mother & her daughter is 20 years . Four years ago , the product of their ages in years was 48.

Solution: Let mother's present age be x years & daughter's present age be (20-x) years.

Four years ago, Mother's age= (x-4) years & daughter's age =(16-x) years

Given,(x-40 (16-x)=48

x^2 -20x+112=0,

After calculation D= -48<0

Therefore no real roots exists .

So the given situation is not possible

Two pipes running together can fill a tank in 6 minutes. If one pipe takes 5 minutes more than the other to fill the tank , find the time in which each pipe would fill the tank separately.

Solution: Suppose the faster pipe takes x minutes to fill at tank

∴.portion of the tank filled by the faster pipe in one minute =1/x

So, portion of the tank filled by the faster pipe in 6 minutes = 6/x

Similarly ,portion of the tank filled by the slower pipe in 6 minutes = 6/(x+5)

ATQ, $6/x + 6/(x+5) = 1$,so, $x^2 - 7x - 30 = 0$

After solving we get x=10 or -3. But x>0 ∴ x = 10

Hence faster pipe fills the tank in 10 minutes & slower pipe takes 10+5 = 15 minutes to fill the tank separately.

THANK YOU